

Note: Section I is compulsory. Attempt any three (3) questions from Section II.

2. Write short answers to any EIGHT questions:

(2 x 8 = 16)

- Show that the parametric equations $x = at^2$, $y = 2at$ represent the equation of parabola $y^2 = 4ax$
- Find $gof(x)$ if $f(x) = \frac{1}{\sqrt{x-1}}$, $x \neq 1$, $g(x) = (x^2 + 1)^2$
- Evaluate $\lim_{x \rightarrow 3} \frac{x-3}{\sqrt{x}-\sqrt{3}}$
- Evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin^2 x}$
- Find the derivative of $\sqrt{x}$ at $x = a$ from first principle.
- Differentiate $\frac{2x-3}{2x+1}$ w.r.t. 'x'.
- Find $\frac{dy}{dx}$ if $y^2 + x^2 - 4x = 5$
- Differentiate $\cos \sqrt{x} + \sqrt{\sin x}$ w.r.t. 'x'.
- Find $\frac{dy}{dx}$ if $y = \log_{10}(ax^2 + bx + c)$
- Find $\frac{dy}{dx}$ if $y = \frac{x}{\ln x}$
- Find y_2 if $y = x^2 \cdot e^{-x}$
- Determine the interval in which $f(x) = \cos x$; $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, is increasing.

3. Write short answers to any EIGHT questions:

(2 x 8 = 16)

- Using differentials find $\frac{dy}{dx}$ and $\frac{dx}{dy}$ for $xy + x = 4$
- Integrate $\frac{1}{\sqrt{x+a} + \sqrt{x}}$ w.r.t. x
- Solve $\int \operatorname{cosec} x \, dx$
- Evaluate $\int \frac{ax}{\sqrt{a^2 - x^4}} \, dx$
- Solve $\int e^{ax} \left[a \sec^{-1} x + \frac{1}{x\sqrt{x^2 - 1}} \right] dx$
- Evaluate $\int_1^2 \frac{x}{x^2 + 2} \, dx$
- Solve $\frac{x^2 + 1}{y + 1} = \frac{x}{y} \frac{dy}{dx}$
- Evaluate $\int_0^{\pi/6} x \cos x \, dx$
- Show that the points $A(0,2)$, $B(\sqrt{3},-1)$ and $C(0,2)$ are vertices of a right triangle.
- Find the equation of line with slope: -5 and y-intercept is -7
- Show that the points $(-1,-3)$, $(1,5)$ and $(2,9)$ lie on the same straight line.
- Find the lines represented by $3x^2 + 7xy + 2y^2 = 0$

(Turn over)

4. Write short answers to any NINE questions:

(2 x 9 = 18)

- Graph the solution set of linear inequality in xy-plane, $3x - 2y \geq 6$
- Find the equation of circle with centre $(\sqrt{2}, -3\sqrt{3})$ and radius $2\sqrt{2}$
- Find the focus and vertex of the parabola $x^2 - 4x - 8y + 4 = 0$
- Write an equation of parabola with axis $y = 0$, through $(2,1)$ and $(11,2)$
- Find the coordinate of vertices of a hyperbola $\frac{y^2}{16} - \frac{x^2}{49} = 1$
- Find the foci of the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$
- Find the sum of vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ given four points $A(1,-1)$, $B(2,0)$, $C(-1,3)$ and $D(-2,2)$
- Find a unit vector in the direction of $\underline{y} = \frac{1}{2}\underline{i} + \frac{\sqrt{3}}{2}\underline{j}$
- Let $\underline{y} = 3\underline{i} - 2\underline{j} + 2\underline{k}$, $\underline{w} = 5\underline{i} - \underline{j} + 3\underline{k}$ find $\underline{y} - 3\underline{w}$
- Find a vector whose magnitude is 4 and is parallel to $2\underline{i} - 3\underline{j} + 6\underline{k}$
- Find the direction cosines of $\overrightarrow{PQ}$ where $P = (2, 1, 5)$, $Q = (1, 3, 1)$
- If $\underline{v}$ is a vector for which $\underline{v} \cdot \underline{i} = 0$, $\underline{v} \cdot \underline{j} = 0$, $\underline{v} \cdot \underline{k} = 0$, find $\underline{v}$
- Find the area of a parallelogram whose vertices are $A(1, 2, -1)$, $B(4, 2, -3)$, $C(6, -5, 2)$, $D(9, -5, 0)$

SECTION II

- Evaluate limit by using algebraic techniques: $\lim_{x \rightarrow a} \frac{x^n - a^n}{x^m - a^m}$ 5
 - Find $\frac{dy}{dx}$ of the given parametric functions: $x = \frac{a(1-t^2)}{1+t^2}$; $y = \frac{2bt}{1+t^2}$ 5
- Show that $\int \sqrt{a^2 - x^2} \, dx = \frac{a^2}{2} \sin^{-1} \frac{x}{a} + \frac{x}{2} \sqrt{a^2 - x^2} + c$ 5
 - Find the area of the region bounded by the triangle whose sides are $7x - y - 10 = 0$, $10x + y - 41 = 0$, $3x + 2y + 3 = 0$ 5
- Solve the given differential equation: $\frac{1}{x} \frac{dy}{dx} = \frac{1}{2}(1 + y^2)$ 5
 - Maximize $f(x,y) = 2x + 5y$ subject to the constraints $2y - x \leq 8$, $x - y \leq 4$; $x \geq 0$, $y \geq 0$ 5
- Find the centre and radius of the circle $4x^2 + 4y^2 - 8x + 12y - 25 = 0$ 5
 - If $\underline{a} + \underline{b} + \underline{c} = 0$, then prove that $\underline{a} \times \underline{b} = \underline{b} \times \underline{c} = \underline{c} \times \underline{a}$ 5
- Show that $y = \frac{\ell n x}{x}$ has maximum value at $x = e$ 5
 - Find the centre, foci and vertices of equation $9x^2 - y^2 - 36x - 6y + 18 = 0$ 5

Roll No. of Candidate _____

Mathematics
Time: 30 Minutes

(INTERMEDIATE PART II)-421-(II)

Code: 8194
OBJECTIVE

GROUP: II
PAPER: II
Marks: 20

Note: You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling of two or more circles will result in zero mark in that question. Attempt as many questions as given in objective type question paper and leave others blank.

- 1- The equation $(x-h)^2 + (y-k)^2 = r^2$ represents
(A) circle (B) hyperbola (C) ellipse (D) parabola
- 2- Vertices of the hyperbola $\frac{x^2}{16} - \frac{y^2}{4} = 1$ are
(A) $(\pm 2, 0)$ (B) $(0, \pm 2)$ (C) $(\pm 4, 0)$ (D) $(0, \pm 4)$
- 3- Volume of parallelepiped with three edges is
(A) $\underline{u} \cdot \underline{v} \times \underline{w}$ (B) $\frac{1}{6} (\underline{u} \cdot \underline{v} \times \underline{w})$ (C) $\frac{1}{3} (\underline{u} \cdot \underline{v} \times \underline{w})$ (D) $\frac{1}{2} (\underline{u} \cdot \underline{v} \times \underline{w})$
- 4- Associated equation of $ax + by > c$ is
(A) $ax + by \geq c$ (B) $ax + by \leq c$ (C) $ax + by = c$ (D) $ax + by < c$
- 5- Normal form of the equation of straight line is
(A) $y - y_1 = m(x - x_1)$ (B) $\frac{x}{a} + \frac{y}{b} = 1$ (C) $x \cos \alpha + y \sin \alpha = p$ (D) $y = mx + c$
- 6- $\int e^{ax} dx$
(A) $e^{ax} + C$ (B) $a e^{ax} + C$ (C) $\frac{e^{ax}}{a} + C$ (D) $e^x + C$
- 7- $\frac{d}{dx} \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right)^2$
(A) $1 - \frac{1}{x^2}$ (B) $1 + \frac{1}{x^2}$ (C) 0 (D) $1 - \frac{1}{2x}$
- 8- If $f(x) = \frac{1}{x}$, then $f^{-1}(x)$ is
(A) 1 (B) x (C) -x (D) $\frac{1}{x}$
- 9- The parabola $x^2 = -4ay$ opens
(A) right side (B) up ward (C) down ward (D) left side

(Turn over)

(2)

- 10- $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos t dt$
(A) $\frac{\sqrt{3}-1}{2}$ (B) $\frac{\sqrt{3}+1}{2}$ (C) $\frac{1-\sqrt{3}}{2}$ (D) $\frac{1+\sqrt{3}}{-2}$
- 11- $\frac{d}{dx} (\cos \sqrt{x})$
(A) $\frac{-\sin \sqrt{x}}{\sqrt{x}}$ (B) $-\sin \sqrt{x}$ (C) $\frac{-\sin \sqrt{x}}{2\sqrt{x}}$ (D) $\frac{\cos \sqrt{x}}{\sqrt{x}}$
- 12- If $f(x) = \frac{x^3-x}{x^2+1}$, then $f(x)$ is
(A) odd function (B) even function (C) linear function (D) quadratic function
- 13- If θ is the angle between two non-zero vectors $\underline{a}, \underline{b}$ then $\sin \theta =$
(A) $\frac{|\underline{a} \times \underline{b}|}{|\underline{a}| |\underline{b}|}$ (B) $\frac{|\underline{a} \cdot \underline{b}|}{|\underline{a}| |\underline{b}|}$ (C) $\frac{\underline{a} \times \underline{b}}{|\underline{a}| |\underline{b}|}$ (D) $\frac{\underline{a} \cdot \underline{b}}{\underline{a} \underline{b}}$
- 14- $\int \ell n \sin x \frac{\cos x}{\sin x} dx$
(A) $\frac{(\ell n \sin x)^2}{2} + c$ (B) $\ell n \sin x + c$ (C) $\ell n (\sin x)^2 + c$ (D) $(\ell n \sin x)^2 + c$
- 15- The distance of the point $(-1, 2)$ from x-axis is
(A) 1 (B) -1 (C) -2 (D) 2
- 16- $\int \frac{1}{x^2} dx$
(A) $x + c$ (B) $\frac{x^3}{3} + c$ (C) $\frac{1}{x} + c$ (D) $\frac{-1}{x} + c$
- 17- Slope of the line $y = 2x + 5$ is
(A) 2 (B) -2 (C) 0 (D) $\frac{1}{2}$
- 18- If $f(x) = \sin x$, then $f'(0)$
(A) 0 (B) -1 (C) 1 (D) undefined
- 19- If $\underline{y} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, then magnitude of $\underline{y}$ is
(A) 29 (B) $\sqrt{29}$ (C) 28 (D) $\sqrt{28}$
- 20- $\frac{d}{dx} \left(\cot^{-1} \frac{x}{a} \right)$
(A) $\frac{-1}{a^2+x^2}$ (B) $\frac{a}{a^2+x^2}$ (C) $\frac{-a}{a^2+x^2}$ (D) $\frac{1}{a^2+x^2}$

Mathematics
Time: 2:30 hours

(INTER PART II)-421

SUBJECTIVE

Note: Section I is compulsory. Attempt any three (3) questions from Section II.

SECTION I

2. Write short answers to any EIGHT questions:

(2 x 8 = 16)

- Determine whether $f(x) = x^{2/3} + 6$ is even or odd function.
- If $f(x) = \sqrt{x+1}$, $g(x) = \frac{1}{x^2}$, $x \neq 0$, find (i) $f \circ g(x)$ (ii) $g \circ f(x)$
- Evaluate $\lim_{x \rightarrow \pi} \frac{\sin x}{\pi - x}$
- Evaluate $\lim_{x \rightarrow \infty} \left(\frac{x}{1+x} \right)^x$
- Find by definition derivative of $(x+4)^{1/3}$
- Differentiate w.r.t. x $\frac{(x^2+1)^2}{x^2-1}$
- Find $\frac{dy}{dx}$ if $xy + y^2 = 2$
- Find $\frac{dy}{dx}$ if $y = x \cos y$
- Find $\frac{dy}{dx}$ if $y = \sin h^{-1}(x^3)$
- Find y_2 if $x^3 - y^3 = a^3$
- Find $\frac{dy}{dx}$ if $y = \frac{x}{\ell n x}$
- Divide 20 into two parts so that the sum of their squares will be minimum.

3. Write short answers to any EIGHT questions:

(2 x 8 = 16)

- Use differentials to find $\frac{dy}{dx}$ if $x^2 + 2y^2 = 16$
- Find $\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx$
- Find $\int \frac{(x+b)}{(x^2+2bx+c)^{1/2}} dx$
- Evaluate $\int \ell n x \, dx$
- Solve $\int e^{2x} [-\sin x + 2 \cos x] dx$
- Evaluate $\int_{-1}^2 (x + |x|) dx$
- Find the area bounded by the curve $y = x^3 + 3x^2$ and the x -axis
- Solve the differential equation $\frac{dy}{dx} = \frac{y}{x^2}$
- Show that the points $A(0, 2)$, $B(\sqrt{3}, -1)$ and $C(0, -2)$ are the vertices of a right triangle.
- Show that the points $(-1, -3)$, $(1, 5)$ and $(2, 9)$ lie on the same straight line.
- Find the equation of line with slope: -5 and y -intercept is -7
- Find the lines represented by $3x^2 + 7xy + 2y^2 = 0$

(Turn over)

GROUP: II
PAPER: II
Marks: 80

(2)

4. Write short answers to any NINE questions:

(2 x 9 = 18)

- Graph the inequality $2x + 3y \leq 18$
- Find radius of the circle $5x^2 + 5y^2 + 24x + 36y + 10 = 0$
- Find centre of the circle $5x^2 + 5y^2 + 14x + 12y - 10 = 0$
- Find the length of the tangent drawn from the point $(-5, 4)$ to the circle $5x^2 + 5y^2 - 10x + 15y - 131 = 0$
- Find focus of the parabola $x^2 = 4(y-1)$
- Find the foci of the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$
- Find the equation of directrices of the hyperbola $\frac{y^2}{16} - \frac{x^2}{9} = 1$
- Find a unit vector in the direction of $\underline{v} = 2\hat{i} + 6\hat{j}$
- Let $\underline{v} = 3\hat{i} - 2\hat{j} + 2\hat{k}$, $\underline{w} = 5\hat{i} - \hat{j} + 3\hat{k}$ find $\underline{v} - 3\underline{w}$
- Find a unit vector perpendicular to the plane containing $\underline{a}$ and $\underline{b}$, where $\underline{a} = 2\hat{i} - 6\hat{j} - 3\hat{k}$ and $\underline{b} = 4\hat{i} + 3\hat{j} - \hat{k}$
- Find the volume of tetrahedron with the vertices $A(2,1,8)$, $B(3,2,9)$, $C(2,1,4)$ and $D(3,3,10)$
- Find the constant α , such that the vectors $\hat{i} - 2\alpha\hat{j} - \hat{k}$, $\hat{i} - \hat{j} + 2\hat{k}$ and $\alpha\hat{i} - 2\hat{j} + \hat{k}$ are coplanar.
- Find the work done, if the point at which the constant force $\underline{F} = 4\hat{i} + 3\hat{j} + 5\hat{k}$ is applied to an object, moves from $P_1(3, 1, -2)$ to $P_2(2, 4, 6)$.

SECTION II

- Evaluate $\lim_{\theta \rightarrow 0} \frac{1 - \cos p\theta}{1 - \cos q\theta}$ 5
 - Prove that $y \frac{dy}{dx} + x = 0$ if $x = \frac{1-t^2}{1+t^2}$; $y = \frac{2t}{1+t^2}$ 5
- Show that $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ell n(x + \sqrt{x^2 - a^2}) + c$ 5
 - Find equations of two parallel lines perpendicular to $2x - y + 3 = 0$ such that the product of the x and y -intercepts of each is 3. 5
- Solve $\int \frac{12}{x^3 + 8} dx$ 5
 - Maximize $f(x, y) = x + 3y$ subject to the constraints $2x + 5y \leq 30$, $5x + 4y \leq 20$; $x \geq 0$, $y \geq 0$ 5
- Find equations of tangent to the circle $x^2 + y^2 = 2$ perpendicular to the line $3x + 2y = 6$ 5
 - Prove that $\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$ 5
- Show that $\cos(x+h) = \cos x - h \sin x - \frac{h^2}{2!} \cos x + \frac{h^3}{3!} \sin x + \dots$ and evaluate $\cos 61^\circ$ 5
 - Find centre, foci and vertices of $\frac{y^2}{16} - \frac{x^2}{9} = 1$ 5

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