

SUBJECTIVE

Note: Section I is compulsory. Attempt any three (3) questions from Section II.

SECTION I

2. Write short answers to any EIGHT questions:

(2 x 8 = 16)

- Determine whether the given function f is even or odd. $f(x) = x^{2/3} + 6$
 - Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{x+a} - \sqrt{a}}{x}$
 - How would you state composition of functions?
 - Express limit in term of e $\lim_{x \rightarrow \infty} \left(\frac{x}{1+x} \right)^x$
 - Give any example of discontinuous function and sketch it.
 - Differentiate w.r.t. 'x'; $\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}$
 - Find $\frac{dy}{dx}$ if $y^3 - 2xy^2 + x^2y + 3x = 0$
 - Find the derivative w.r.t variable involved; $\cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$
 - Find $f'(x)$ if $f(x) = \ln(\sqrt{e^{2x} + e^{-2x}})$
 - Produce y_2 from $y = e^x \sin x$
 - Determine the interval in which f is increasing $f(x) = \cos x$, $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2} \right)$
 - Find the dimensions of a rectangle of the largest area having perimeter 120 centimeters.
3. Write short answers to any EIGHT questions: (2 x 8 = 16)
- Using differentials find $\frac{dy}{dx}$ and $\frac{dx}{dy}$ in the equation $x^2 + 2y^2 = 16$
 - Evaluate the integral $\int \frac{x}{x+2} dx$
 - Find $\int \frac{dx}{x(\ln 2x)^3}$
 - Evaluate the integral $\int \frac{1}{(1+x^2) \tan^{-1}x} dx$
 - Evaluate the integral $\int e^{-x}(\cos x - \sin x) dx$
 - Evaluate the definite integral $\int_{-6}^2 \sqrt{3-x} dx$
 - Find the area above the x-axis and under the curve $y = 5 - x^2$ from $x = -1$ to $x = 2$
 - Solve the differential equation $\frac{dy}{dx} = \frac{y}{x^2}$
 - Find the distance and the mid-point of the line joining two points $A(-\sqrt{5}, -\frac{1}{3})$, $B(-3\sqrt{5}, \sqrt{5})$
 - In triangle $A(8, 6)$, $B(-4, 2)$, $C(-2, 6)$ find the slope of each side of the triangle.
 - Find an equation of each of the lines represented by $20x^2 + 17xy - 24y^2 = 0$
 - Find area of triangular region whose vertices are $A(5, 3)$, $B(-2, 2)$, $C(4, 2)$

(Turn over)

(2)

4. Write short answers to any NINE questions:

(2 x 9 = 18)

- Graph the inequality $2x + 1 \geq 0$
- What is the optimal solution?
- Write the general equation of circle having centre at origin.
- Write the equation of circle with centre $(-3, 5)$ and radius 7.
- Write the parametric equations of an ellipse.
- Find the centre and foci of ellipse $25x^2 + 9y^2 = 225$
- Discuss and sketch the graph of equation $25x^2 - 16y^2 = 400$
- Find the point of intersection of conics $x^2 + y^2 = 8$, $x^2 - y^2 = 1$
- Find the value of $2\mathbf{i} \times 2\mathbf{j} \cdot \mathbf{k}$
- Find $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$ If $\mathbf{a} = 3\mathbf{i} - \mathbf{j} + 5\mathbf{k}$, $\mathbf{b} = 4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and $\mathbf{c} = 2\mathbf{i} + 5\mathbf{j} + \mathbf{k}$
- Find $\mathbf{a}$ so that $|\mathbf{a}\mathbf{i} + (\mathbf{a} + 1)\mathbf{j} + 2\mathbf{k}| = 3$
- Write any two properties of dot product.
- Find the cosine of the angle θ between the vectors: $\mathbf{u} = 3\mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$

SECTION II

- (a) If $y = \frac{x\sqrt{a+x}}{\sqrt{a-x}}$ then find $\frac{dy}{dx}$ 5

(b) Evaluate $\lim_{\theta \rightarrow 0} \frac{\tan \theta - \sin \theta}{\sin^3 \theta}$ 5
- (a) Evaluate $\int \frac{\sqrt{2}}{\sin x + \cos x} dx$ 5

(b) Find equations of two parallel lines perpendicular to $2x - y + 3 = 0$ such that the product of the x and y-intercepts of each is 3. 5
- (a) Find $\int_0^{\pi/4} (1 + \cos^2 \theta) \tan^2 \theta d\theta$ 5

(b) Maximize $f(x, y) = x + 3y$ subject to constraints: 5

$$2x + 5y \leq 30, \quad 5x + 4y \leq 20, \quad x \geq 0, \quad y \geq 0$$
- (a) Find an equation of the line through the intersection of $16x - 10y - 33 = 0$, $12x + 14y + 29 = 0$ and the intersection of $x - y + 4 = 0$, $x - 7y + 2 = 0$ 5

(b) Write an equation of the circle that passes through the points $A(4, 5)$, $B(-4, -3)$, $C(8, -3)$ 5
- (a) Find centre, foci, eccentricity, vertices and equations of directrices of hyperbola 5

$$\frac{(y+2)^2}{9} - \frac{(x-2)^2}{16} = 1$$

(b) Find two vectors of length 2 parallel to the vector $\mathbf{v} = 2\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$ 5