

MATHEMATICS
Time: 2:30 hours

Intermediate Part II Class 12th (1st A 423)

GROUP: I
PAPER: II
Marks: 80

SUBJECTIVE

Note: Section I is compulsory. Attempt any three (3) questions from Section II.

SECTION I

2. Write short answers to any EIGHT questions: (2 x 8 = 16)

- Show that the parametric equations $x = a \cos \theta$, $y = b \sin \theta$ represent the equation of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
- Let the real valued functions 'f' and 'g' be defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 1$, obtain the expressions $\log(x)$ and $f^{-2}(x)$
- Evaluate the limit $\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x^2}$
- Differentiate w.r.t.x $\frac{2x-1}{\sqrt{x^2+1}}$
- Find $\frac{dy}{dx}$ if $x = at^2$ and $y = 2at$
- Find $\frac{dy}{dx}$ if $4x^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$
- If $\tan y(1 + \tan x) = 1 - \tan x$, show that $\frac{dy}{dx} = -1$
- Find y_2 if $y = \ln \left(\frac{2x+3}{3x+2} \right)$
- Determine the intervals in which f is increasing or decreasing for the domain mentioned.
 $f(x) = \sin x$; $x \in (-\pi, \pi)$
- Find two positive integers whose sum is 30 and their product will be maximum.
- Define feasible region and feasible solution.
- Graph the feasible region of the following system of linear inequalities and find the corner points
 $x + y \leq 5$
 $-2x + y \geq 2$
 $x \geq 0$

3. Write short answers to any EIGHT questions: (2 x 8 = 16)

- Find δy if $y = x^2 - 1$ and x changes from 3 to 3.02
- Evaluate $\int \frac{(1-\sqrt{x})^2}{\sqrt{x}} dx$
- Find the anti-derivative of $x^2 \ln x$
- Evaluate $\int \frac{e^{m \tan^{-1} x}}{1+x^2} dx$
- Evaluate $\int_{\pi/6}^{\pi/3} \cos t dt$
- Find the area between x-axis and the curve $y = \sin 2x$ from $x = 0$ to $x = \frac{\pi}{3}$
- Solve the differential equation $\frac{dy}{dx} = \frac{1+y^2}{e^{-x}}$
- If $\vec{v} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{w} = 5\hat{i} - \hat{j} + 3\hat{k}$ then find $|3\vec{v} + \vec{w}|$

(Turn over)

(2)

- Find direction cosines of vector $\vec{PQ}$ where $P(2,1,5)$ and $Q(1,3,1)$
- Find a vector perpendicular to each of the vectors $\vec{u} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{v} = 4\hat{i} + 2\hat{j} - \hat{k}$
- Prove that $\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b}) = 0$
- Calculate the projection of $\vec{a} = \hat{i} - \hat{k}$ along $\vec{b} = \hat{j} + \hat{k}$

4. Write short answers to any NINE questions: (2 x 9 = 18)

- Find the point three-fifth of the way along the line segment from $A(-5, 8)$ to $B(5, 3)$
- By means of slopes show that the points $(-4, 6)$, $(3, 8)$ and $(10, 10)$ lie on the same line.
- Find an equation of line with x-Intercept $= -9$ and slope is -4
- Find measure of angle between the lines represented by $10x^2 - 23xy - 5y^2 = 0$
- Find h such that the points $A(-1, h)$, $B(3, 2)$ and $C(7, 3)$ are collinear.
- Find an equation of the line through $(1, -5)$ and parallel to a line with slope -24 .
- Find the co-ordinates of the point that divides the join of $A(-6, 3)$ and $B(5, -2)$ externally in ratio 2:3
- Find centre and radius of the circle $4x^2 + 4y^2 - 8x + 12y - 25 = 0$
- Write down an equation of the parabola with focus $(2, 5)$ and directrix $y = 1$
- Find an equation of circle of radius a and lying in 2nd Quadrant such that it is tangent to both the axes.
- Find focus, vertex of the parabola $x^2 = 4(y-1)$
- Find an equation of the hyperbola with given foci $(0, \pm 6)$, $e = 2$
- Find centre and foci of the hyperbola $\frac{y^2}{4} - x^2 = 1$

SECTION II

Note: Attempt any three (3) questions.

- Express the limit in terms of e $\lim_{x \rightarrow 0} \frac{e^{1/x} - 1}{e^{1/x} + 1}$, $x > 0$ 5
 - Find $\frac{dy}{dx}$ of the parametric equations $x = \frac{a(1-t^2)}{1+t^2}$, $y = \frac{2bt}{1+t^2}$ 5
- Show that $\int \sqrt{a^2 - x^2} dx = \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} \right) + \frac{x}{2} \sqrt{a^2 - x^2} + c$ 5
 - Find an equation of the line through the point $(2, -9)$ and intersection of the lines $2x + 5y - 8 = 0$ and $3x - 4y - 6 = 0$ 5
- Evaluate $\int_0^{\pi/4} \cos^4 t dt$ 5
 - Maximize $f(x, y) = 2x + 5y$ subject to the constraints $2y - x \leq 8$; $x - y \leq 4$; $x \geq 0$; $y \geq 0$ 5
- If $y = (\cos^{-1} x)^2$, prove that $(1 - x^2)y_2 - xy_1 - 2 = 0$ 5
 - Write down an equation of the circle that passes through the given points $A(-7, 7)$, $B(5, -1)$, $C(10, 0)$ 5
- Find centre, foci, eccentricity, vertices and directrices of $x^2 + 16x + 4y^2 - 16y + 76 = 0$ 5
 - Prove that in any $\triangle ABC$; $a = b \cos C + c \cos B$ 5

312-1stA 423-23000

Roll No. of Candidate _____

MATHEMATICS
Time: 30 Minutes

Intermediate Part-II, Class 12th (1stA 423- I)
OBJECTIVE
Code: 8192

GROUP: II
PAPER: II
Marks: 20

Note: You have four choices for each objective type question as A, B, C and D. The choice which you think is correct, fill that circle in front of that question number. Use marker or pen to fill the circles. Cutting or filling of two or more circles will result in zero mark in that question.

- 1- 1- If $f(x) = \sqrt{x-12}$, then $f(16) =$
(A) 16 (B) 12 (C) 28 (D) 2
- 2- $\lim_{x \rightarrow 3} (2x+4) =$
(A) 3 (B) 6 (C) 10 (D) 12
- 3- If $y = \ln(\sin x)$, then $\frac{dy}{dx} =$
(A) $\tan x$ (B) $\cot x$ (C) $-\tan x$ (D) $-\cot x$
- 4- $\frac{d}{dx} \left(\frac{2}{x} \right) =$
(A) $\ln|x^2|$ (B) $-\frac{2}{x^2}$ (C) $-2x^2$ (D) 2^x
- 5- If $y = \cosh 2x$, then $\frac{dy}{dx} =$
(A) $2\sinh 2x$ (B) $-\sinh 2x$ (C) $-2\sinh 2x$ (D) $\cosh 2x$
- 6- If $f(x) = \sin x$, then $f(\pi) =$
(A) -1 (B) 1 (C) 0 (D) $\frac{1}{2}$
- 7- $\int_0^{\pi/2} \cos x \, dx =$
(A) 2 (B) 0 (C) -1 (D) 1
- 8- $\int \tan x \, dx =$
(A) $\ln \cos x$ (B) $\ln |\sec x|$ (C) $\ln \sin x$ (D) $\ln |\cot x|$
- 9- $\int \sqrt{x} \, dx =$
(A) $\frac{\sqrt{x}}{2}$ (B) $\frac{x\sqrt{x}}{3}$ (C) $\frac{1}{2\sqrt{x}}$ (D) $\frac{2x\sqrt{x}}{3}$
- 10- $\int \sin 2x \, dx =$
(A) $-\frac{\cos 2x}{2}$ (B) $\frac{\cos 2x}{2}$ (C) $2 \cos 2x$ (D) $-2 \cos 2x$
- 11- If a line is parallel to x-axis, then inclination =
(A) 0° (B) 30° (C) 45° (D) 90°
- 12- If a line intersects y-axis at $(0, a)$, then 'a' is called
(A) x-intercept (B) y-intercept (C) inclination (D) slope

(Turn over)

(2)

- 13- $y = mx + c$ is _____ form of equation of line.
(A) normal (B) point-slope (C) slope-intercept (D) intercept
- 14- $x = 0$ is the solution of inequality
(A) $3x - 2 > 0$ (B) $3x + 5 < 0$ (C) $2x - 6 < 0$ (D) $x + 3 < 0$
- 15- Two circles are said to be concentric if they have same _____.
(A) radius (B) diameter (C) center (D) length
- 16- Mid-point of hypotenuse of a right triangle is called as
(A) circumcentre (B) incentre (C) orthocentre (D) centroid
- 17- The focus of parabola $y^2 = 4ax$ is
(A) $(0, a)$ (B) $(-a, 0)$ (C) $(a, 0)$ (D) $(0, -a)$
- 18- The co-ordinates of vertex of parabola $x + 8 - y^2 + 2y = 0$ will be
(A) $(-9, 1)$ (B) $(9, 1)$ (C) $(9, -1)$ (D) $(-9, -1)$
- 19- $\cos 0 =$
(A) $\hat{a} \cdot \hat{b}$ (B) $|a \times b|$ (C) $a \times b$ (D) $\sin 0$
- 20- $\hat{i} \cdot (2\hat{j} \times \hat{k}) =$
(A) 0 (B) 2 (C) 4 (D) 6

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MATHEMATICS
Time: 2:30 hours

Intermediate Part-II, Class 12th (1stA 423)

SUBJECTIVE

Note: Section I is compulsory. Attempt any three (3) questions from Section II.

SECTION I

2. Write short answers to any EIGHT questions:

(2 x 8 = 16)

- Prove that $\operatorname{sech}^2 x = 1 - \operatorname{Tanh}^2 x$
- Evaluate $\lim_{x \rightarrow 3} \frac{x-3}{\sqrt{x}-\sqrt{3}}$
- Find $\lim_{x \rightarrow \pi} \frac{\sin x}{\pi - x}$
- If $y = x^4 + 2x^2 + 2$, prove that $\frac{dy}{dx} = 4x\sqrt{y-1}$
- Differentiate $\sin x$ w.r.t $\cot x$
- If $y = \cot^{-1}\left(\frac{x}{a}\right)$, find $\frac{dy}{dx}$
- If $f(x) = \ln(e^x + e^{-x})$, find $f'(x)$
- If $y = \operatorname{Tanh}^{-1}(\sin x)$, find $\frac{dy}{dx}$
- If $y = \sqrt{x} + \frac{1}{\sqrt{x}}$, find y_2
- Find the interval in which $f(x)$ is increasing, $f(x) = 4 - x^2$, $x \in (-2, 2)$
- Define problem constraints.
- Graph the solution set of linear inequality in xy-plane, $3x + 7y \geq 21$

3. Write short answers to any EIGHT questions:

(2 x 8 = 16)

- Using differentials find $\frac{dy}{dx}$, if $xy + x = 4$
- Evaluate $\int (2x+3)^{1/2} dx$
- Evaluate $\int \frac{\sec^2 x}{\sqrt{\tan x}} dx$
- Evaluate $\int e^{-x} (\cos x - \sin x) dx$
- Evaluate $\int_{-1}^2 (x + |x|) dx$
- Find the area bounded by Cos function from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$
- Solve $\frac{dy}{dx} = \frac{y}{x^2}$
- If O is the origin and $\overrightarrow{OP} = \overrightarrow{AB}$, find the point P when A and B are (-3,7) and (1,0) respectively.
- Find a unit vector in the direction of $\underline{v} = \underline{i} + 2\underline{j} - \underline{k}$
- Find α so that $\underline{u}$ and $\underline{v}$ are perpendicular $\underline{u} = 2\alpha\underline{i} + \underline{j} - \underline{k}$ and $\underline{v} = \underline{i} + \alpha\underline{j} + 4\underline{k}$
- Find a unit vector perpendicular to the plane containing $\underline{a}$ and $\underline{b}$, where $\underline{a} = 2\underline{i} - 6\underline{j} - 3\underline{k}$, $\underline{b} = 4\underline{i} + 3\underline{j} - \underline{k}$
- Given a force $\vec{F} = 2\underline{i} + \underline{j} - 3\underline{k}$ acting at a point A(1,-2,1). Find the moment of $\vec{F}$ about the point B(2,0,-2)

(Turn over)

GROUP: II
PAPER: II
Marks: 80

(2)

4. Write short answers to any NINE questions:

(2 x 9 = 18)

- Show that the points A(0,2), B($\sqrt{3}$, -1) and C(0,-2) are vertices of a right triangle.
- The two points P(3,2) and O'(1,3) are given in XY-coordinate system. Find the XY-coordinates of P referred to the translated axes O'X and O'Y
- Find K so that the line joining A(7,3), B(K,-6) and the line joining C(-4,5), D(-6,4) are parallel.
- Find an equation of the vertical line through (-5,3)
- Find the distance from the point P(6,-1) to the line $6x - 4y + 9 = 0$
- Find point of intersection of the lines $x - 2y + 1 = 0$ and $2x - y + 2 = 0$
- Find measure of the angle between the lines represented by $x^2 - xy - 6y^2 = 0$
- Find an equation of the circle with centre at (5,-2) and radius 4
- Check the position of the point (5,6) with respect to the circle $x^2 + y^2 = 81$
- Find the focus and vertex of parabola $x^2 = -16y$
- Find equation of ellipse with foci ($\pm 3, 0$) and minor axis of length 10
- Find the centre and foci of $x^2 - y^2 = 9$
- Find the point of intersection of the given conics $x^2 + y^2 = 8$ and $x^2 - y^2 = 1$

SECTION II

Note: Attempt any three (3) questions.

- If $f(x) = \begin{cases} \frac{\sqrt{2x+5} - \sqrt{x+7}}{x-2}, & x \neq 2 \\ k, & x = 2 \end{cases}$, Find value of k so that f is continuous at $x = 2$
 - Differentiate $\cos x^2$ from the first principle.
- Evaluate $\int e^{2x} \cos 3x dx$
 - Find the area of the region bounded by the triangle with vertices (a,b+c), (a,b-c) and (-a,c)
- Solve the differential equation $y - x \frac{dy}{dx} = 2 \left(y^2 + \frac{dy}{dx} \right)$
 - Minimize $z = 2x + y$ subject to constraints $x + y \geq 3$, $7x + 5y \leq 35$, $x \geq 0$, $y \geq 0$
- If $x = a(\theta + \sin \theta)$, $y = a(1 + \cos \theta)$ then, show that $y^2 = \frac{d^2 y}{dx^2} + a = 0$
 - Find an equation of the circle which passes through the points A(5,10), B(6,9) and C(-2,3)
- Find an equation of the ellipse with centre (0,0), major axis horizontal, the points (3,1), (4,0) lie on the graph.
 - Find the volume of the tetrahedron whose vertices are A(2,1,8), B(3,2,9), C(2,1,4) and D(3,3,10)

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